Constraint satisfaction

- Units represent hypotheses about parts of a problem
- Weights code constraints on how hypotheses can combine (i.e., the degree to which they are consistent or inconsistent)
- Possible solutions correspond to particular patterns of active units
- External input introduces bias to favor one possible solution over others

Example: Map coloring

Assign colors to regions so that no adjacent regions have the same color.

Hypotheses
- An assignment of a color to a region

Constraints
- Adjacent regions must be assigned different colors
- Only one color can be assigned to each region
- Each region must be assigned a color

Biases
- Initial color preference for a given region
Maximizing Goodness (= minimizing Energy)

Global measure of degree to which activations satisfy weight constraints

\[ G (\text{Goodness} = -\text{Energy}) = \sum_{i<j} a_i a_j w_{ij} \]

How should unit \( k \) behave locally so as to increase global Goodness?

- Set \( a_k = 1 \) if \( G_{a_k=1} > G_{a_k=0} \) (or, equivalently, \( G_{a_k=1} - G_{a_k=0} > 0 \))

\[
G_{a_k=1} = \sum_i a_i w_{ik} + \sum_{i<j \neq k} a_i a_j w_{ij}
\]

\[
G_{a_k=0} = \sum_{i<j \neq k} a_i a_j w_{ij}
\]

- Set \( a_k = 1 \) if \( \sum_i a_i w_{ik} > 0 \) (= binary threshold unit)

Sigmoid units: increase activation as net input increases (monotonicity)

Goodness surface in state space

State space

- A high-dimensional space with a dimension for each of \( n \) units in the network
- Each unit’s activity (state) can be interpreted as a coordinate along its corresponding axis/dimension
- At any instant in time, the current pattern of activity over the entire network corresponds to a particular \( n \)-dimensional point in the space
- As units update their states, the point moves in state space

Goodness surface

- Each instantaneous pattern of activity has a corresponding “goodness” value
- Add an additional \((n+1)\)st dimension to state space so that the goodness of each point (activity pattern) can be plotted “above” it
- The set of goodness values corresponding to all possible activity patterns forms a continuous surface “above” state space

Goodness surface (Necker cube)
Goodness surface with input to front face (left cube)